An Efficient FEM Formulation for Rotationally Symmetric Coaxial Waveguides

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***Abstract*— In this paper, an efficient finite element method formulation (FEM) is applied to rotationally symmetric coaxial waveguides, in which the dependence of the dominant TEM mode on the radius of the cylindrical coordinate system is explicitly taken into account in the basis functions. In this way, a physically appropriate approximation of the unknown field distribution is achieved. After scaling, the resulting sparse matrix equation is solved iteratively by using the biconjugate gradient method (BCG). The numerical results show excellent agreement with results of the mode matching technique (MMT). Compared with the conventional FEM formulation, this method yields a significant improvement in accuracy within the frequency range where the TEM mode dominates.**

I. INTRODUCTION

THE COAXIAL transmission line is one of the most frequently used waveguides for transporting electromagnetic energy within the microwave range. While the theory of the homogeneous line is well established, discontinuities and inhomogeneous coaxial waveguides still require some attention. The efforts date back as early as 1944, when Whinnery and co-workers studied discontinuities in coaxial waveguides with the mode matching technique [1]. Recently, these problems were tackled with purely numerical methods, e. g. the finite-difference time-domain (FDTD) [2] and the finite element method (FEM) ([3]–[5]). Satisfactory results can be achieved with these methods.

In the following, an efficient FEM formulation is introduced for solving rotationally symmetric coaxial waveguide problems. In contrast to the conventional FEM formulation, where a transformation of the unknown field distribution is utilized in order to eliminate the quasi-singular term in the variational formulation, we try to include maximal available knowledge about the unknown field distribution already in the Ansatz. The basis functions are chosen to take explicitly into account the quasi-singular behavior of the dominant TEM-mode in coaxial waveguides. In this way, the unknown field distribution can appropriately be approximated. As a result, this formulation leads to a more efficient solution than the conventional one.

Similar approaches have already found wide application with approximate solutions of Maxwell's equations. For example, special basis functions are used in the FEM to enforce the edge condition [6]. In the asymptotic analysis of the field distribution in the neighborhood of a caustic, an expansion including the Airy function and its derivative, which appear in the asymptotic expansion of the exact solution of the corresponding canonical problem, has also proven advantageous [7, Ch. 3]. The work reported here was encouraged by the authors' experience with a similar method, which has successfully been applied to the quality factor optimization of coaxial resonators [8]. While entire-domain basis functions were used in that former study, owing to their flexibility sub-domain basis functions will be utilized in this paper.

In Section II, our approach is described in some detail. For the purpose of comparison the conventional formulation is given as well. Numerical results are presented in Section III together with results from the conventional FEM formulation. All numerical results are validated by using a mode matching program developed at our institute several years ago [9]. Finally, the paper is summarized in Section IV.

II. FINITE ELEMENT FORMULATION

The geometry of a rotationally symmetric inhomogeneous coaxial waveguide transition, e. g. a connector between two homogeneous coaxial waveguides, is depicted in Fig. 1. The z-axis of the circular cylindrical coordinate system (r, φ, z) is chosen to coincide with the axis of the coaxial waveguide. The region Ω in the zr-plane is bounded by its boundary Γ, which consists of the walls of the transition Γ_0 and the apertures of the two homogeneous waveguides Γ_1 and Γ_2. In a source free region, the first two time-harmonic Maxwell equations are given below

$$\text{curl}\ \mathbf{H} = j\omega\epsilon\mathbf{E}, \tag{1}$$

$$\text{curl}\ \mathbf{E} = -j\omega\mu\mathbf{H}, \tag{2}$$

where $\mathbf{E}$ ($\mathbf{H}$) is the electric (magnetic) field strength. Permittivity ϵ and permeability μ can both be complex and position-dependent in the region of interest Ω. $\omega = 2\pi f$ is the angular frequency. The harmonic time factor $e^{j\omega t}$ is suppressed in the rest of this paper.

Scalar-multiplying the equation which results after eliminating the electric field strength $\mathbf{E}$ from (1) with the help of (2), with a vectorial weighting function $\mathbf{W}$, and then integrating the resulting equation in a source free volume, where Ω is the cross section of this volume with a half plane $\varphi =$ const., yields

$$\int_\Omega j\omega\mu\mathbf{H}\cdot\mathbf{W}r dS + \int_\Omega \frac{\text{curl}\ \mathbf{H}\cdot\text{curl}\ \mathbf{W}}{j\omega\epsilon} r dS - \int_\Gamma \hat{n}\cdot(\mathbf{E}\times\mathbf{W}) r dl = 0; \tag{3}$$

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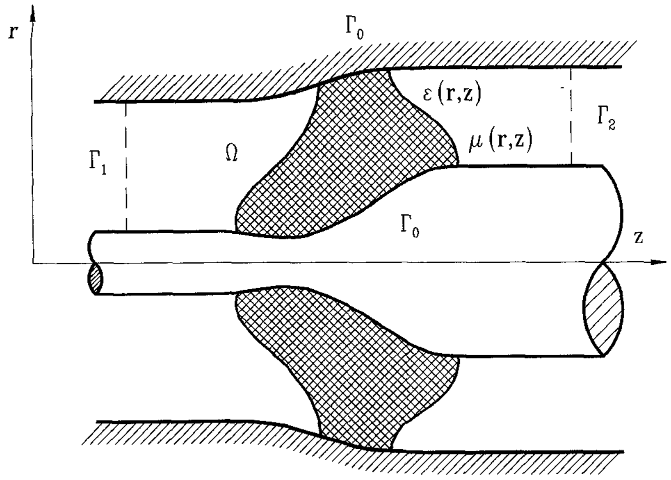


Fig. 1. Geometry of a rotationally symmetric inhomogeneous coaxial waveguide

In the above derivation, the relation div $(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\text{curl }\mathbf{A} - \mathbf{A}\cdot\text{curl }\mathbf{B}$ is already taken into account. $\hat{n}$ is the inward normal vector of the boundary Γ.

A similar equation can be derived for the electric field strength $\mathbf{E}$ in an analogous way. But (3) will be prefered in this paper, due to the simple form of the resulting equation, as shown in what follows.

In (3) there are two unknown functions, namely $\mathbf{E}$ and $\mathbf{H}$. But the unknown $\mathbf{E}$ appears in the integral over the boundary Γ only. Clearly, other conditions, e. g. boundary conditions must be considered in order to make (3) solvable.

On Γ_0, the walls of the waveguide, which are assumed to be perfectly conducting in this study, the tangential components of the electric field must disappear. Owing to rotational symmetry, the only existing component of the magnetic field strength in the circular cylindrical coordinate system is $\mathbf{H} = H_\varphi\hat{\varphi}$, where $\hat{\varphi}$ is the unit vector of coordinate φ. With this in mind and with the logical choice $\mathbf{W} = W_\varphi\hat{\varphi}$, (3) can be simplified to

$$\int_\Omega j\omega\mu H_\varphi W_\varphi r dr dz + \int_\Omega \frac{\text{curl }(H_\varphi\hat{\varphi})\cdot\text{curl }(W_\varphi\hat{\varphi})}{j\omega\epsilon} r dr dz - \sum_{i=1}^{2}\int_{\Gamma_i}\hat{n}\cdot(\mathbf{E}\times(W_\varphi\hat{\varphi}))r dl = 0; \quad (4)$$

Equation (1) yields:

$$\mathbf{E} = \frac{\text{curl }(H_\varphi\hat{\varphi})}{j\omega\epsilon} = \frac{1}{j\omega\epsilon}\left(-\frac{\partial H_\varphi}{\partial z}\hat{r} + \frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi)\hat{z}\right). \quad (5)$$

In addition, the following relations on the aperture Γ_1 with $z = z_1$ $(< z_2)$ exist:

$$\hat{n} = \hat{z}, \quad (6)$$

$$H_\varphi = H_\varphi^i + H_\varphi^s, \quad (7)$$

$$\frac{\partial H_\varphi^i}{\partial z} = -jk_1 H_\varphi^i, \quad (8)$$

$$\frac{\partial H_\varphi^s}{\partial z} = jk_1 H_\varphi^s; \quad (9)$$

For the aperture Γ_2 with $z = z_2$

$$\hat{n} = -\hat{z}, \quad (10)$$

$$H_\varphi = H_\varphi^s, \quad (11)$$

$$\frac{\partial H_\varphi^s}{\partial z} = -jk_2 H_\varphi^s. \quad (12)$$

k_i is the wavenumber on aperture $\Gamma_i (i = 1, 2)$. It is assumed that apertures Γ_1 and Γ_2 are so distant to the inhomogeneity, that all higher order modes are heavily attenuated and hence can be negleted. This advantageous way of incorporating the boundary conditions on apertures of homogeneous waveguides is due to Williamson, Lee and Mittra [4].

With the help of the above equations, (4) can be further simplified to

$$\int_\Omega \frac{1}{j\omega\epsilon}\left[\frac{\partial H_\varphi}{\partial z}\frac{\partial W_\varphi}{\partial z} + \frac{1}{r^2}\frac{\partial}{\partial r}(rH_\varphi)\frac{\partial}{\partial r}(rW_\varphi)\right] r dr dz + \int_\Omega j\omega\mu H_\varphi W_\varphi r dr dz + \frac{k_1}{\omega\epsilon}\int_{\Gamma_1} H_\varphi W_\varphi r dr + \frac{k_2}{\omega\epsilon}\int_{\Gamma_2} H_\varphi W_\varphi r dr = 2\frac{k_1}{\omega\epsilon}\int_{\Gamma_1} H_\varphi^i W_\varphi r dr \quad (13)$$

Evidently, the $\frac{1}{r}$ term appears in the integrand of the above equation. It is exactly this term which makes the construction of triangular finite element universal matrices impossible. To avoid this, the following substitution is made [10]:

$$H_\varphi = \sqrt{r}h, \quad (14)$$

$$W_\varphi = \sqrt{r}w. \quad (15)$$

This transforms (13) into

$$\int_\Omega \frac{1}{j\omega\epsilon}\left[r^2\left(\frac{\partial h}{\partial z}\frac{\partial w}{\partial z} + \frac{\partial h}{\partial r}\frac{\partial w}{\partial r}\right) + \frac{3r}{2}\left(h\frac{\partial w}{\partial r} + w\frac{\partial h}{\partial r}\right) + \frac{9hw}{4}\right] dr dz + \int_\Omega j\omega\mu r^2 hw dr dz + \frac{k_1}{\omega\epsilon}\int_{\Gamma_1} r^2 hw dr + \frac{k_2}{\omega\epsilon}\int_{\Gamma_2} r^2 hw dr = 2\,\frac{k_1}{\omega\epsilon}\int_{\Gamma_1} H_\varphi^i(\sqrt{r}\,)^3 w dr. \quad (16)$$

which is applied to axisymmetrical coaxial discontinuity problems in [3]–[5].

Clearly, the term $\frac{1}{r}$ now no longer exists and the position-independent matrices result with the exception of the integral at the right hand side of (16), albeit this integral can be evaluated analytically. But unfortunately this transformation has no physical meaning: in a region including the axis, it leads to solutions containing the term $\sqrt{r}$ near the axis, as pointed out by Daly [11]. He proposed an alternative substitution instead, which gives physically appropriate solutions in regions *including* the axis. But for coaxial problems, where the axis is not included, both these substitutions are not suitable [8]: it is well known that the radial dependence of all the field components of the TEM mode in a homogeneous coaxial waveguide is given exactly by the quasi-singular term $\frac{1}{r}$ [12]. Even for φ-independent higher order E-modes, which will normally be excited with the problems under investigation, there exists $H_\varphi \sim \frac{1}{r}$ for $r \to 0$. This relation can be verified easily by applying the limiting forms of the Bessel functions for small values of the argument [13, p. 513] to the exact expressions of the corresponding modes [12, pp. 72–80].

In order to obtain a physically meaningful solution, it seems appropriate to introduce an alternative substitution where the r-dependence of the TEM mode in a homogeneous coaxial waveguide is taken into account explicitly, namely [8],

$$h = rH_\varphi, \tag{17}$$

$$w = rW_\varphi. \tag{18}$$

From the above considerations, the final equation for the numerical calculation takes the following form

$$\int_\Omega \left[j\omega\mu hw + \frac{1}{j\omega\epsilon}\left(\frac{\partial h}{\partial z}\frac{\partial w}{\partial z} + \frac{\partial h}{\partial r}\frac{\partial w}{\partial r}\right)\right]\frac{1}{r}drdz + \frac{k_1}{\omega\epsilon}\int_{\Gamma_1} hw\frac{1}{r}dr + \frac{k_2}{\omega\epsilon}\int_{\Gamma_2} hw\frac{1}{r}dr = 2\frac{k_1}{\omega\epsilon}\int_{\Gamma_1} H_\varphi^i w dr. \tag{19}$$

The construction of position-independent universal matrices is not possible with this formulation. Its application is warranted by the clear physical meaning of the Ansatz and the achieved high accuracy of the numerical results (see the following section), though. Additionally, position independent universal matrices do not exist in general, if isoparametric elements [14] are used, as is done in this paper.

In the application of this substitution to the quality factor optimization of coaxial resonators [8], high order polynomials were used as entire-domain basis functions. This choice is especially suitable for convex or slightly concave regions [15], [16]. The assumption is justified by the resulting optimal shape of coaxial resonators [8]. Contrary, sub-domain basis functions, namely the isoparametric elements are used in this paper, because more complex regions can be treated efficiently and the numerically resulting linear dependence in case of entire-domain basis functions can completely be eliminated [17].

In this study, linear and quadratic triangular and quadrilateral isoparametric elements are used. Inside each element, the unknown function h is substituted by a series of polynomial interpolation functions together with the unknown values of the function h at some nodes. The weighting function is chosen to be identical with the interpolation function (Galerkin method).

All the integrals are evaluated numerically by using Gaussian quadrature [18]. Due to the symmetry of the element matrix, only half of the entries are stored using the so called clique storage [19]. After scaling, the resulting matrix equation is solved iteratively by utilizing the biconjugate gradient method [20]. The symmetry of the resulting matrix leads to a much faster convergence than the conjugate gradient method.

In the following section, this formulation will be compared with the conventional FEM formulation and with an available program COAXIAL which was developed at our institute by using the standard mode matching technique [9].

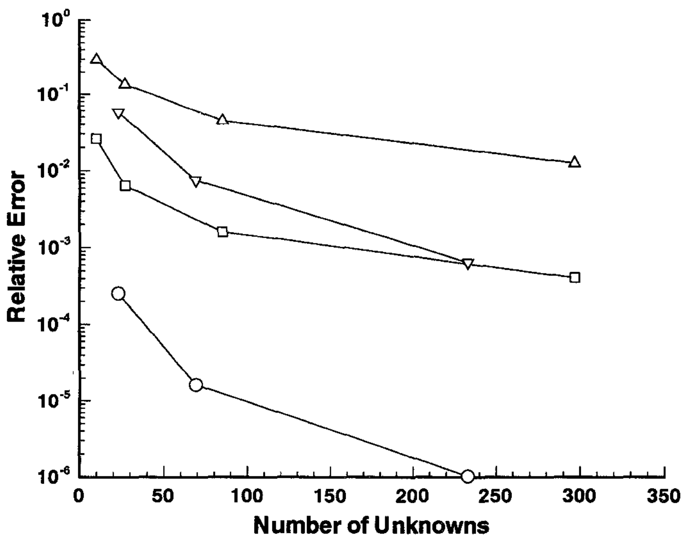


Fig. 2. Relative error of the dominant resonant frequency calculated by using different formulations and different discretization (□— □— □: linear quadrilaterals / new FEM; ○—○—○: quadratic quadrilaterals / new FEM; △—△—△: linear quadrilaterals / conventional FEM; ▽—▽—▽: quadratic quadrilaterals / conventional FEM)

III. Numerical Examples

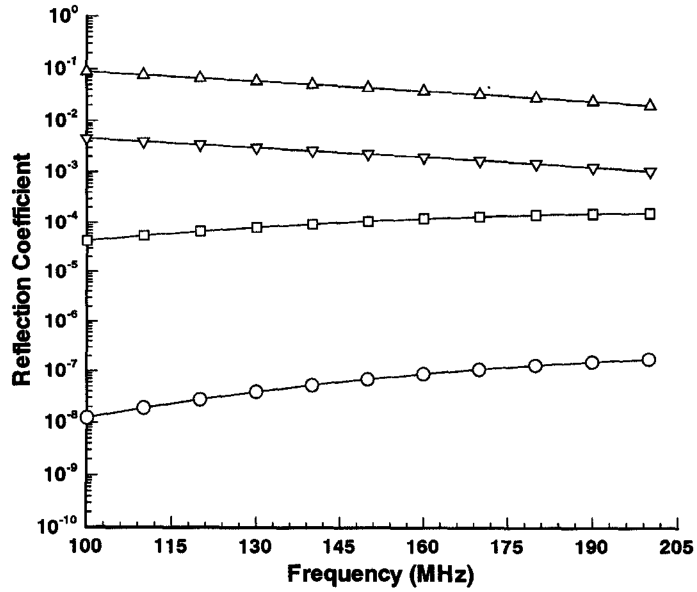
For the following numerical calculations utilizing the two FEM formulations discussed above, the *same* mesh discretization is used for each example. For the calculation of matrix elements, Gaussian quadratures of identical accuracy are used in both cases. A comparison between (16) and (19) shows that the numerical implementation of the former is clearly more expensive than that of the latter. Owing to the absence of the $\frac{1}{r}$ term, the matrix elements of (16) can be calculated more accurately than the matrix elements in (19). If the biconjugate gradient method is used to solve the resulting matrix equation, a comparable number of iterations is necessary for both the conventional and the one proposed in this paper for a given accuracy of the residuum.

A. Coaxial Resonators

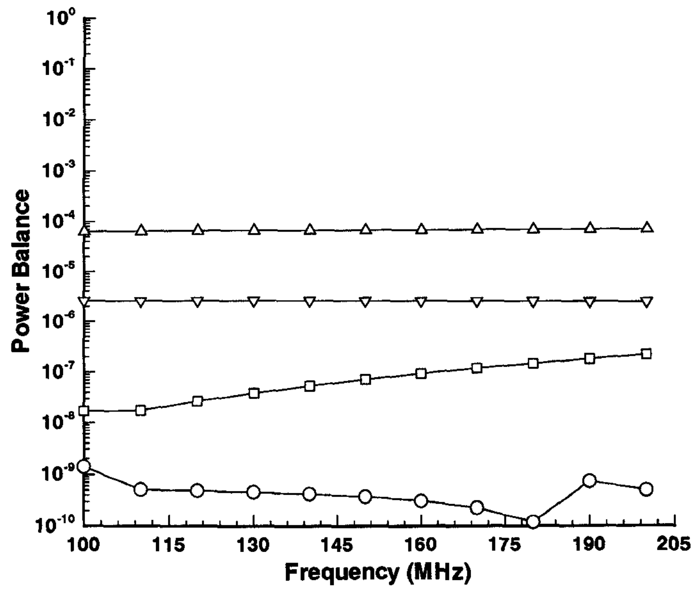
The first example to be considered is a coaxial resonator. It consists of a homogeneous coaxial waveguide short-circuited at both ends. The resonance frequencies of this kind of resonator can be determined analytically. Owing to this factor, this kind of resonators is very suitable for comparison purposes.

The coaxial resonator to be dealt with has a radius of the outer conductor of 0.23 m and a radius of the inner conductor of 0.1 m. The length of the resonator is 0.52 m. Its geometry is discretized with linear and quadratic quadrilaterals. The relative error of the numerical results of the eigenvalue corresponding to the dominant mode is depicted in Fig. 2, as a function of the number of unknowns and as a function of the order of the used elements.

As expected, the efficient formulation is clearly superior to the conventional one. This fact can be observed by the first few eigenvalues. The well known fact that higher accuracy



(a)



(b)

Fig. 3. Reflection coefficient (a) and power balance (b) of a coaxial waveguide with incident TEM mode at one end (□— □— □: 512 linear triangles / new FEM; ○— ○—○: 128 quadratic triangles / new FEM; △—△—△: 512 linear triangles / conventional FEM; ▽—▽—▽: 128 quadratic triangles / conventional FEM)

can be achieved by using higher order elements is confirmed in that figure as well.

B. Coaxial Waveguide

The second example treats a coaxial waveguide with the same dimension as the resonator in example 1. But here the two ends are terminated with the characteristic impedance of the waveguide and a TEM mode is incident from one end. For the discretization linear and quadratic triangular elements are utilized.

Fig. 3 depicts the amplitude of the reflection coefficient and the power balance as a function of the frequency. In reality, no reflection exists. Due to the inability to describe the TEM mode properly, the results of the conventional formulation are not very satisfactory. The superiority of the efficient FEM formulation to the conventional one is demonstrated again.

C. Compensated Dielectric Support

The above examples are analytically solvable and the exact waves are strict transverse electromagnetic (TEM). As a result, the superiority of the new FEM formulation to the conventional one is expected. But can this advantage be retained for more complex geometries? This question will be answered now by the application of the two FEM formulations to a compensated dielectric support. Its geometry is shown in the insert of Fig. 4 and is discretized by using linear and

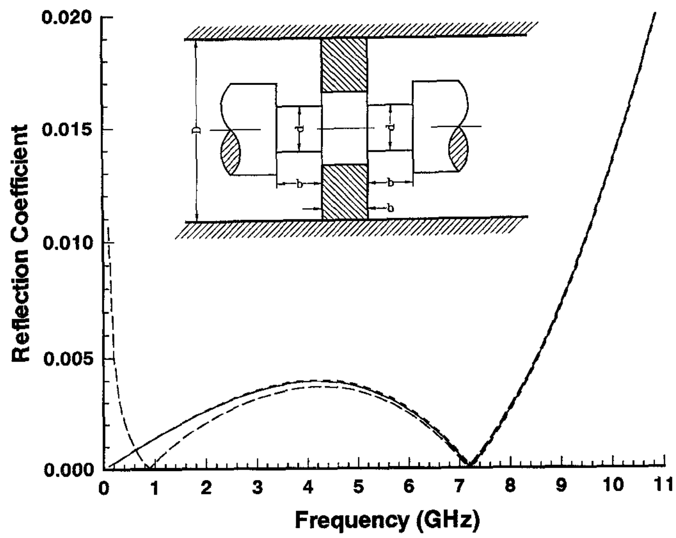


Fig. 4. Reflection coefficient of a compensated dielectric support for a 60 Ω coaxial line with $D = 10$ mm, $b = 1$ mm, $d = 0.1725D$ and the relative permittivity $\epsilon_r = 2.55$ (———: 2624 quadratic quadrilaterals / new FEM: - - - - -: mode matching technique with 20 modes; — — —: 2624 quadratic quadrilaterals / conventional FEM)

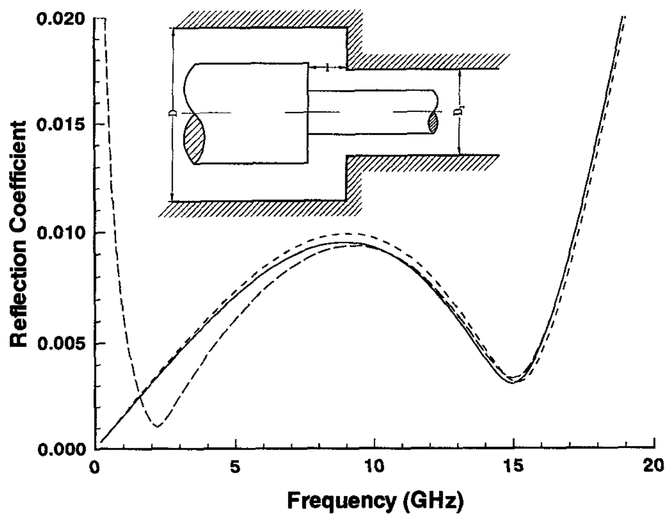


Fig. 5. Reflection coefficient of an optimal transition between two 60 Ω coaxial lines with $D = 10$ mm, $D_1 = 3.623$ mm and $l = 0.129D$ (———: 2460 quadratic quadrilaterals / new FEM; - - - - -: mode matching technique with 20 modes; — — —: 2460 quadratic quadrilaterals / conventional FEM)

quadratic quadrilaterals. While this problem cannot be solved analytically, the mode matching technique [21] can be applied very conveniently.

Owing to the fact that as many mode functions as necessary can be included in the mode matching technique, this method is clearly far superior to both FEM formulations mentioned in this paper, where they are applicable. At the same time, an unavoidable disadvantage of the semi-analytical methods is evident: only a limited number of problems can be solved.

To begin with, this example is firstly solved with the mode matching technique, by using the program COAXIAL [9]. Twenty modes are included to achieve numerical convergence. Then the geometry of this example is discretized by utilizing 2624 (8161 unknowns) quadratic quadrilaterals. The numerical results of the efficient FEM formulation is shown in Fig. 4 as well. To achieve comparable accuracy, 10496 linear quadrilaterals with 10785 unknowns must be used in the new formulation. A very good agreement between these results can be observed. The results of the conventional formulation with 2624 second order quadrilaterals are also given in Fig. 4. From 0.1 to about 4 GHz, this result deviates evidently from the other ones.

D. Optimal Transition

Similar deviations can be observed in another example, which consists of an optimal transition between two coaxial waveguides with different geometrical dimensions but the same characteristic impedance (60 Ω).

The mode matching technique is applicable to this geometry (Fig. 5). Again 20 modes are used. The geometry is descritized in 2460 quadratic quadrilaterals with 7679 unknowns. While the numerical results of the new FEM formulation agree very well with that of the mode matching technique, the numerical results of the conventional FEM show a clear deviation from the other two results (Fig. 5). The large difference observed in this example is probably due to the presence of the high impedance coaxial waveguide between the two 60 Ω ones.

The last two examples were calculated by using the FDTD method as given in [2]. The results there show a clear similarity with the ones presented in this paper by using the mode matching technique and the new FEM. A possible explanation for the existing deviation between the FDTD and the results reported here could stem from the relatively coarse descritization used in [2].

IV. Conclusion

A physically appropriate, and as a result, computationally efficient finite element formulation is introduced in this paper to solve rotationally symmetric coaxial waveguide discontinuities or transitions. Compared with the conventional finite element method, the new formulation results in evidently higher accuracy and demands less computation time. This is expected, because the principal behavior of the field distribution in such waveguides is already described properly in the Ansatz. In some sense, this formulation has some similarities with the well known model problem method in the asymptotic analysis of high frequency diffraction problems.

The advantages of this formulation are demonstrated with the help of several examples. The numerical results have been verified by exact results, where possible, and by numerical results of a mode matching technique program. In cases where the mode matching technique is applicable, this method should preferably be used compared with purely numerical methods, to which the proposed formulation in this paper belongs as well. But for more complex problems, such as continuous transitions, or if the losses of the waveguide walls are to be considered, then purely numerical methods must be utilized instead. It is always desirable and sometimes possible to make purely numerical methods more efficient, as shown in this paper.

This idea can be incorporated into the numerical calculation of similar problems as well, e. g., the calculation of noncircular coaxial waveguides. It will be an interesting task to improve the efficiency of other numerical methods by using similar ideas.

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